Theorem **Intermediate Value Theorem**

Let f be a continuous function on [a, b] and, without loss of generality, let f(a) < f(b). Then for every value y, where f(a) < y < f(b), there is at least one value c in (a, b) such that f(c) = y.

Example : 1

Let f be continuous on [1, 5] where f(1) = −2 and f(5) = −10. Does a value 1 < c < 5 exist such that f(c) = −9? Why/why not ?

Given that $f(x)$ is continuous on $[1,5]$ and $f(1) = -2$, $f(5) = -10$

We know that $f(x)$ goes from -2 to -10 on $[1,5]$ since -9 lies b/w -2 and -10

MATHWISE Tutors
Your Path to Math Success
https://www.mathwisetutors.com/

So By Intermediate value Theorem

There must exist at least one value c in interval $[1,5]$ such that $f(c) = -9$

So Yes, there Exist c such that $f(c) = -9$

Example: 2

Let g be continuous on [−3, 7] where g(0) = 0 and g(2) = 25. Does a value −3 < c< 7 exist such that g(c) = 15? Why/why not?

Given that $g(x)$ is continuous on $[-3,7]$, $g(0)=0$, $g(2)=25$

We know that $g(x)$ goes from 0 to 25 on $[-3,7]$

since 15 lies b/w 0 and 25

So By Intermediate value Theorem

There must exist at least one value c in interval $[-3,7)$ such that $g(c)=15$

Example: 3

Let f be continuous on [−1, 1] where f(−1) = −10 and f(1) = 10. Does a value −1 < c < 1 exist such that f(c) = 11? Why/why not?

As f is continous on $[-1,1]$ and $f(-1)=-10$, $f(1)=10$

$f(x)$ goes from -10 to 10. But 11 doesn't lies b/w -10 and 10

So By Intermediate value theorem

There is no such c exist such that $f(c)=11$

To learn more topics

MATHWISE Tutors
Your Path to Math Success

Youtube Channel: https://www.youtube.com/channel/UCCKak3MAQDGLZZtNgTQW9pQ

Facebook page Link: https://web.facebook.com/mathwisetutor/?_rdc=1&_rdr

Instagram page Link: https://www.instagram.com/mathwisetutor/?hl=en

Website Link: https://www.mathwisetutors.com