Function and inverse function

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- ☑ Domain and Range
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- ☑ Identity function

Function

➢ Definition :

A function ***f*** **: A → B** is a rule that assigns one, and only one, element of B to each element of A.

We may therefore view ***f*** as a collection of ordered pairs in **A × B**:

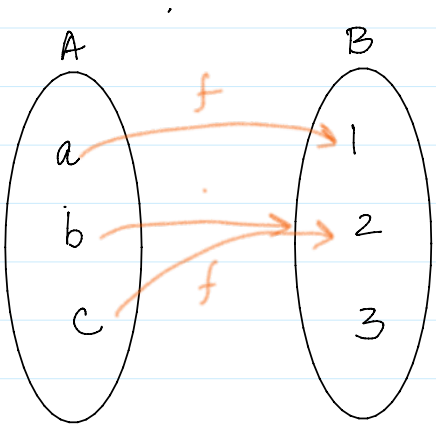
$$\{ (a, f(a)) : a \in A \}$$

➢ Domain and Range :

Let **R ⊆ A × B** be a relation from A to B. The domain and range of R are the sets

$$\text{dom}(R) = \{a \in A : (a, b) \in R \text{ for some } b \in B\},$$

$$\text{range}(R) = \{b \in B : (a, b) \in R \text{ for some } a \in A\}.$$



$f(a) = 1$ $f(c) = 2$

$f(b) = 2$

$\text{dom}(f) = A$

Note That :

A function from A to B is a relation $f \subseteq A \times B$ satisfying the following conditions:

- $\text{dom}(f) = A$,
- $(a, b_1), (a, b_2) \in f \Rightarrow b_1 = b_2$

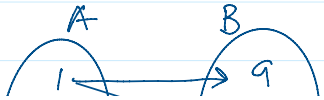
⇒ Every element of A is related to only one element of B

➢ Examples :

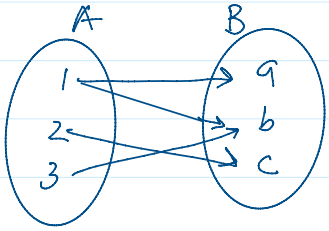
1. If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$

Is relation $R \subseteq A \times B = \{(1, a), (1, b), (2, c), (3, b)\}$ a function?

As 1 is related to a and b both

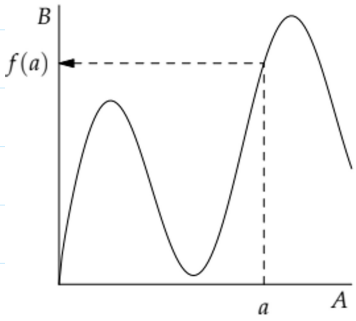


As 1 is related to a and b both domain (f) is repeating So it is not a function

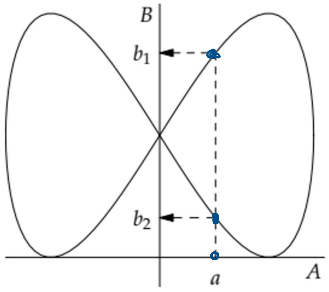


2. Which of the followings is function?

(a)



(b)



(a) It is a function

(b) It is not a function because $f(a) = b_2$ and $f(a) = b_1$ and $b_1 \neq b_2$

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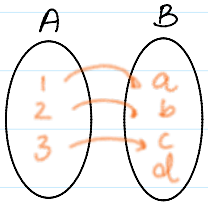
➢ Types of Functions :

- **One-one function / Injective:**

A function $f:A \to B$ is called injective (or one-to-one) if it maps distinct elements of the domain A to distinct elements of the codomain B. In other words, f is injective if:

$$\forall a_1, a_2 \in A, \quad f(a_1) = f(a_2) \Rightarrow a_1 = a_2$$

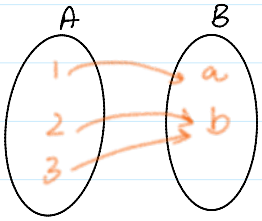


- **Onto function / Surjective :**

A function $f:A \to B$ is called surjective (or onto) if every element of the codomain B is the image of at least one element of the domain A. In other words, f is surjective if:

$$\forall\, b \in B, \exists\, a \in A \text{ such that } f(a) = b$$

This means that the range of f is the entire codomain B.



Rang (f) = B

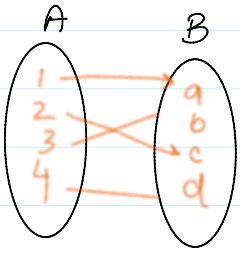
- **One-one onto function / Bijective :**

A function $f:A \to B$ is called bijective if it is both injective and surjective. This means that f maps each element of A to a unique element of B, and every element of B is mapped by some element of A. In other words, f is bijective if:

A function f is called bijective if it is both injective and surjective. This means that f maps each element of A to a unique element of B, and every element of B is mapped by some element of A. In other words, f is bijective if:

$\forall\, b \in B, \exists!\, a \in A$ such that $f(a) = b$

The symbol $\exists!$ means "there exists exactly one."



Dom(f) = A

Range(f) = B

No element is repeating its image

The inverse of a Function

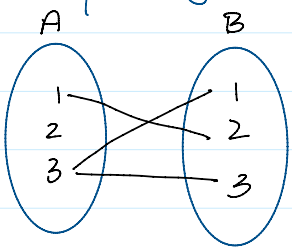
The inverse of a function $f \subseteq A \times B$ is the inverse relation $f^{-1} \subseteq B \times A$.

$\text{dom}(f^{-1}) = \text{range}(f)$ and $\text{range}(f^{-1}) = \text{dom}(f)$.

➢ Example :

Let A = B = {1, 2, 3} and consider the relation f = {(1, 3),(2, 1),(3, 3)}.

then $f^{-1} = \{(3,1)(1,2),(3,3)\}$ is not a function because domain elements are repeating.



➢ Identity Function :

Let A be any set and define a relation $id_A : A \to A$ by

$Id_A = \{(a, a) : a \in A\}$.

Then id is a bijective function called the identity function on A.

Inverse of Identity function is also a bijective function

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